Exercise 2

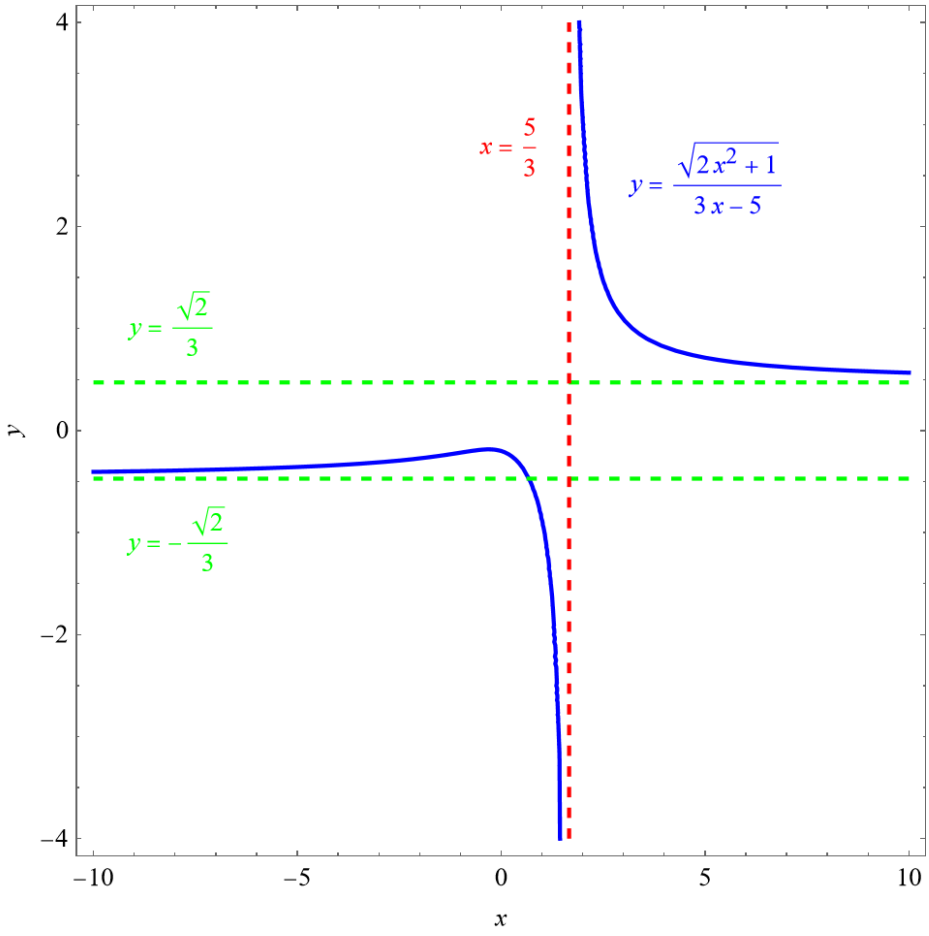
(a) Can the graph of $y = f(x)$ intersect a vertical asymptote? Can it intersect a horizontal asymptote? Illustrate by sketching graphs.

(b) How many horizontal asymptotes can the graph of $y = f(x)$ have? Sketch graphs to illustrate the possibilities.

Solution

Part (a)

The graph of $y = f(x)$ cannot intersect a vertical asymptote, but it can intersect a horizontal asymptote.



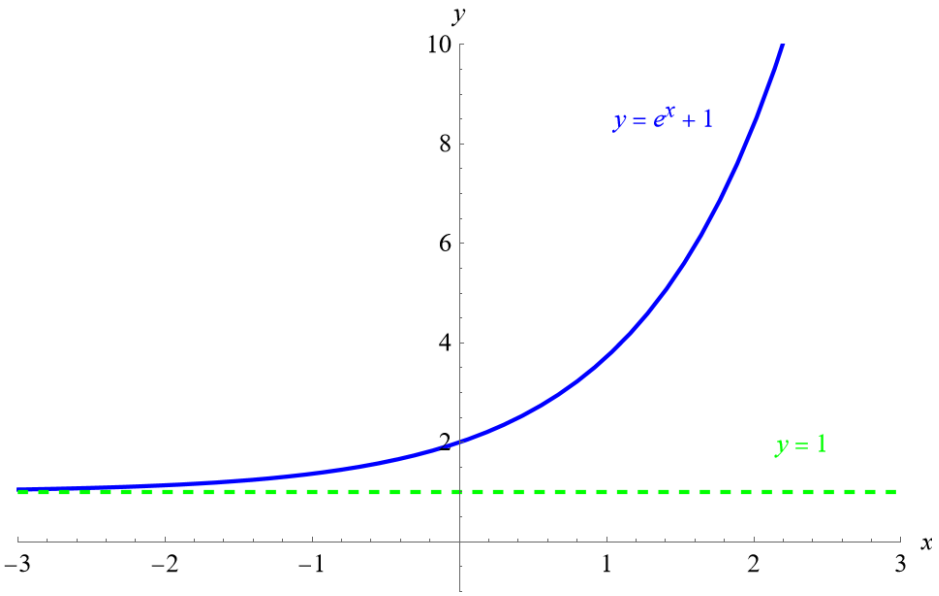
Part (b)

There can be zero, one, or two horizontal asymptotes, depending if the limits,

$$\lim_{x\to-\infty} f(x) = L_- \quad \text{and} \quad \lim_{x\to\infty} f(x) = L_+,$$

are finite and unique.

Below is an example of a graph with a finite limit on only one side.



Below is an example of a graph with finite and identical limits.